Application of Benfor's equations to the problem of "seeing through layers"

Georgi P. Krivoshiev,[a] Raina P. Chalucova[a] and Donald J. Dahm[b]

[a] *Institute for Horticulture and Canned Foods – 154, V. Aprilov Blvd, Plovdiv-4000, Bulgaria*

[b] *TechMan Enterprises, 90 Chapin Greene Dr., Ludlow, MA 01056, USA*

Introduction

The V-method for non-destructive measurement of the internal transmittance of objects, was developed in 1996 by G. Krivoshiev.[1] The internal transmittance is obtained by correcting the overall transmittance spectrum with the diffuse reflectance spectrum. The method was used successfully in the sorting of fruits and vegetables according to their internal quality. There was an increase in the accuracy of assessment, because the contribution of the peel to the spectrum was eliminated without the products being peeled physically. Because of this, the method was called the V (virtual) method. (A detailed description of the method was published by Krivoshiev, Chalucova and Moukarev in the journal *LWT*[2] and Krivoshiev reported it during the 10th International Diffuse Reflectance Conference—IDRC-2000 inChambersburg, Pensilvania, USA. The applicability of the V-method for on-line sorting of potato tubers was presented by the same authors in *NIR news*[3] and also published as a poster at IDRC-2000 and an abridged version of the report.)

The method is based on the hypothesis that the objects can be treated as a three-layer structure according to Figure 1—two external layers, A and B and an internal layer, O. The total transmittance for this structure may be modelled using the following approximation

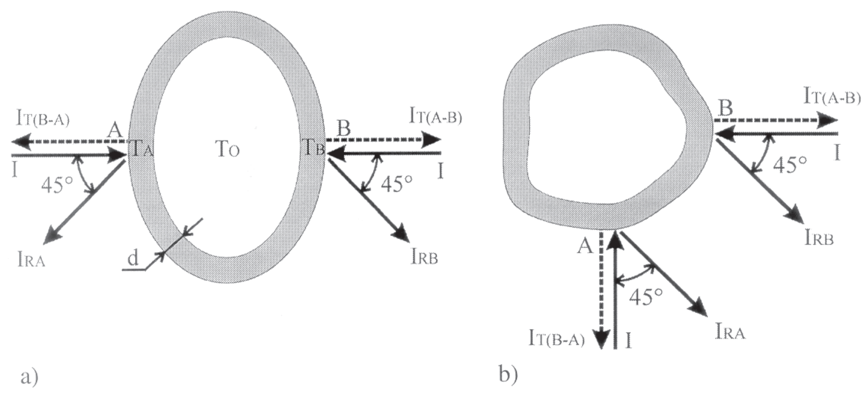

Figure 1. Schemes illustrating the procedure for the nondestructive measurement of the real product flesh transmittance T_O: (a) direct transmittance geometry ($T_{0/180}$) and (b) geometry $T_{0/90}$ — incident and reflected light flux; —- scattered/transmitted/emitted light flux.

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$$T = T_A.T_O.T_B \tag{1}$$

which would be exact for plane-parallel layers from a non-scattering material. In formula (1), T is the overall transmittance of the object, and T_A, T_O and T_B are the transmittances of the layers A, B and O, respectively.

Since the external (surface) layers A and B have only one accessible side, their transmittances are measured by using the relationship known as the Kubelka–Munk (K–M) theory.[4]

$$T_{A(B)} = \frac{b}{ashbSd + bchbSd} = \frac{(1 - R_\infty^2)\exp(-Sd)}{1 - R_\infty^2 \exp(-2bSd)} \tag{2}$$

where: R is the diffuse reflectance from a layer with a thickness large enough so that a further thickness increase does not have an effect on the reflectance; Sd is the scattering power (S: scattering coefficient, cm^{-1}; d: average length of the light path, cm);

$$a = \frac{(1 + R_\infty^2)}{2R_\infty} \qquad \text{and} \qquad b = \frac{(1 - R_\infty^2)}{2R_\infty}$$

Substituting formula (2) in formula (1) and after straightforward transformations, Krivoshiev *et al.* (2000) and Krivoshiev (2000) have proposed two basic models for measuring the internal optical density. The first model is:

$$\begin{aligned} OD_O &= a_0 + a_1 \,\mathrm{In}(1/T) + a_2 \,In\left(1 - R_A^2\right) + a_3 \,\mathrm{In}\left(1 - R_B^2\right) + \\ &+ a_4 \,\mathrm{In}\left[1 - R_A \bullet \exp(-2b_A \bullet (Sd)_A\right] + a_5 \,\mathrm{In}\left[1 - R_B^2 \bullet \exp(-2b_B \bullet (Sd)_B)\right] + \\ &+ a_6 b_A (Sd)_A + a_7 b_B (Sd)_B \end{aligned} \tag{3(a)}$$

The second model

$$OD_O = a_0 + a_1 \,\mathrm{In}\ (1/T) + a_2 \,\mathrm{In}\ (1/R_A) + a_3 \,\mathrm{In}\ (1/R_B) \tag{3(b)}$$

is based on the first one and is obtained by assuming that $R^2\exp(-2bSd) << 1$ and that there is high correlation between the values $(1- R^2)$ and b on the one hand and $\ln(1/R)$ on the other hand. The models are optimised empirically.

The model [3(a)] contains the scattering power Sd of the surface layers, the value of which is unknown. In the V-method, this difficulty is overcome by setting Sd to a constant "average" value $\overline{Sd}$, which is obtained empirically for a given wave range and for a particular type of object. $\overline{Sd}$ is also included in the coefficients of the model [3(b)]. In the present work, this approximation, as well as that represented by the use of formula (1), is eliminated. The modelling is based not on the K–M theory, but the theory of discontinuum by using equations of Benfor.[5,6]

The hypothesis

Applying the Benfor equations[5,6] to the two-layer structure A + O, shown on Figure 1, we obtain

$$T_{A+O} = \frac{T_A \bullet T_O}{(1 - R_{A.} \bullet R_O} \tag{4}$$

$$R_{A+O} = \frac{T_A^2 R_O}{(1 - R_{A.} \bullet R_O)} + R_A \tag{5}$$

According to formula (5) the transmittance of the surface layer, A, can be expressed by using the reflectances

$$T_A = \sqrt{\frac{(R_{A+O} - R_A)(1 - R_A \bullet R_O)}{R_O}} \tag{6}$$

It is obtained analogously for T_B

$$T_B = \sqrt{\frac{(R_{B+O} - R_B)(1 - R_B \bullet R_O)}{R_O}} \tag{7}$$

The three-layer structure A + O + B can be presented as a two-layer one—(A + O) + B. According to equations (4) and (5) the overall transmittance in the direction A → B will be

$$T = T_{(A+O)+B} = \frac{T_{A+O} \bullet T_B}{(1 - R_{A+O} \bullet R_B)} \tag{8}$$

and the reflectance from side A.

$$R_{(A+B)+B} = R_{A+O} + \frac{T_{A+O}^2 \bullet T_B}{(1 - R_{A+O} \bullet R_B)} \tag{9}$$

By replacing formulae (4), (5) and (6) into formula (8) we obtain the overall transmittance.

$$T = \frac{T_A \bullet T_O \bullet T_B}{(1 - R_A \bullet R_O)(1 - R_{A+O} \bullet R_B)} \tag{10}$$

The inclusion of the denominator in formula (10) removes the approximation included in formula (1). However, the values in the denominator cannot be measured non-destructively.

If we replace formulae (6) and (7) into equation (10), the expression of the overall transmittance will be based entirely on the Benfor's equations

$$T = T_O \frac{\sqrt{(R_{A+O} \bullet R_A)(1 - R_A \bullet R_{OA})(R_{B+O} - R_B)(1 - R_B \bullet R_{OB})}}{\sqrt{R_{OA} \bullet R_{OB}(1 - R_A \bullet R_{OA})(1 - R_{A+O} \bullet R_B)}} \tag{11}$$

It is obvious that T_O is a function of seven parameters, from which only three parameters—T, R_{A+O}, R_{B+O}—can be measured instrumentally without the destruction of the object. We assume additionally that:

(a) $R_{A+O} \approx R_{A\infty}$ and $R_{B+O} \approx R_{B\infty}$ because the thickness of the intermediate layer is sufficiently big;
(b) the reflectance R_O of the internal layer O is marked with two indices, A and B, in order to underline that the object inside is not homogenous as a matter of principle, thus the inside reflectance in the directions A → B and B → A is not equal, i.e. $R_{OA} \neq R_{OB}$. Therefore, the theory of discontinuum could be applied only in cases when R_A, R_O and R_B are *a priori* known.

The analysis of equation (5) and the pure physical considerations indicate that if, $R_{A+O} >> R_{OA}T^2_A$, $R_A \approx R_{A+O}$. It seems that for the objects, approximately complying with that requirement (such as fruits, vegetables, eggs, etc.) a reasonable solution for practical utilisation of equation (10) is the transmittances T_A and T_B, may be obtained from the K-M equation (2). By replacing formula (2) for T_A and T_B into formula (10) and by putting $R_A \approx R_{A+O} = R_{A\infty}$, $R_B \approx R_{B+O} = R_{B\infty}$ $R^2e^{-2bSd} << 1$, we obtain

$$T = T_O \frac{(1 - R_A^2)(1 - R_B^2)\exp(-b_A(Sd))_A - b_B(Sd)_B}{(1 - R_A R_{OA})(1 - R_A R_B)} \tag{12}$$

Hence it is easy to obtain the regression model for the determination of the internal optical density

$$OD_O = \ln(1/T_O) = a_0 + a_1 \ln(1/T) + a_2 \ln(1-R_A^2) + a_3 \ln(1-R_B^2) + \qquad [13(a)]$$
$$+ a_4 b_A + a_5 b_B + a_6 \ln\left(1 - R_A \overline{R_{OA}}\right) + a_7 \ln(1 - R_A R_B)$$

R_{OA} is the average reflectance, which is obtained empirically. A simplified version of this model is

$$OD_O = \ln(1/T_O) = a_0 + a_1 \ln(1/T) + a_2 \ln(1-R_A) + a_3 \ln(1-R_B) + \qquad [13(b)]$$
$$+ a_4 \ln\left(1 - R_A \overline{R_{OA}}\right) + a_5 \ln(1 - R_A R_B)$$

Conclusion

From the comparison of formulae (3) and (13), it would seem that the last two terms of formula (13) should yield higher accuracy of the OD_O prediction than formula (3), but this is a question that must be answered experimentally for the specific case. It may be presumed that for objects with a great degree of varying internal transmittance it is more suitable to use formula (3) and for objects with a great degree of varying reflectance of the surface layer, formula (13).

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