

# The costs of sampling errors and bias to the mining industry

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This is Richard Minnitt's regular contributing presentation at WCSB8, Perth. This presentation is another example of a subject matter that lends itself eminently to graphic rendition, which is the exact reason it was decided also to bring this feature in its original PowerPoint format; the presentation layout and style is overwhelmingly pleasing. There is here a wealth of information regarding an issue which is often lamented as lacking: what are the economic costs of inferior sampling. Richard Minnitt here collects a range of illustrative examples that will serve well for all samplers trying to convey the everlasting message: "It pays to pay attention to unnecessary sampling errors and—bias".

## The costs of sampling errors and bias to the mining industry

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### Its all about the money

- Research into the theory and practice of particulate sampling, 1850 to 1930, motivated by incorrect payments for traded commodities in USA and UK
- Substantial financial implications and scale of potential losses through poor sampling
- Sampling errors and bias highlighted the need for accurate assays of ores, concentrates, and coals
- Researchers knew of errors and bias but did not specify source

### Prolific authors and writers

- Huge body of research - Sharwood and von Bernewitz (1922, US Bureau of Mines) 906 pieces of literature sampling of ores and concentrates
- Other researchers were Reed (1882), Brunton (1895), Hofman (1899), Warwick (1903), Rickard (1907), Richards (1908), Argall (1912)
- Henry Vezin, practical sampling expert wrote very little, but in 1850 he designed and published diagrams of his rotary sampler
- Vezin's design implies he understood principles of probabilistic and correct sampling, namely "each and every fragment must have the same statistical opportunity as every other fragment of being in the sample"
- From the 1950s onwards Gy (2004) developed what is called the Theory of Sampling (TOS)



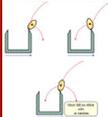
The Vezin Sampler

### Source and nature of sampling errors

Sources of error	Name of error	Nature of error
Material characterization	In-situ Nugget Effect (INE)	True error
	Fundamental Sampling Error (FSE)	True error
	Grouping and Segregation Error (GSE)	Bias
Sampling equipment and materials handling	Delineation Error (DE)	Bias
	Extraction Error (EE)	Bias
	Preparation Error (PE)	Bias
	Weighing Error (WE)	Bias
Plant process and procedures	Continuous Selection Error (CSE)	Error and Bias
Analytical processes	Analytical Error (AE)	Error and bias

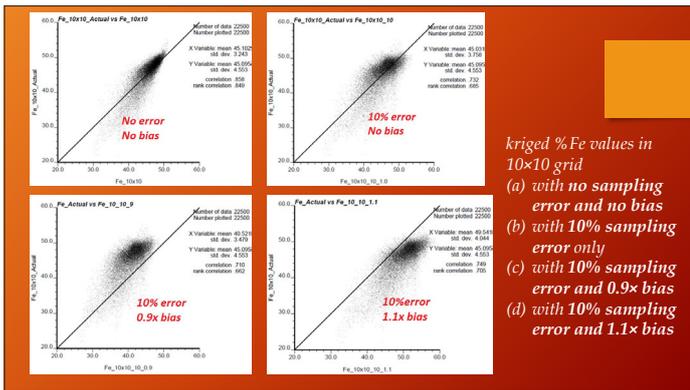
### Sampling bias different from sampling error

- Sampling bias generated by interactions at the interface between steel of sampling tools and broken ores
- Biased sampling occurs when particles in the lot, because of size, shape, density, or moisture content, are consistently favoured over others in the sampling process; "...each and every fragment does not have the same statistical chance of being in the sample."
- Sampling bias can be engineered out of sampling equipment provided we comply with:
- 1) principle of Symmetry and 2) principle of the Centre of Gravity

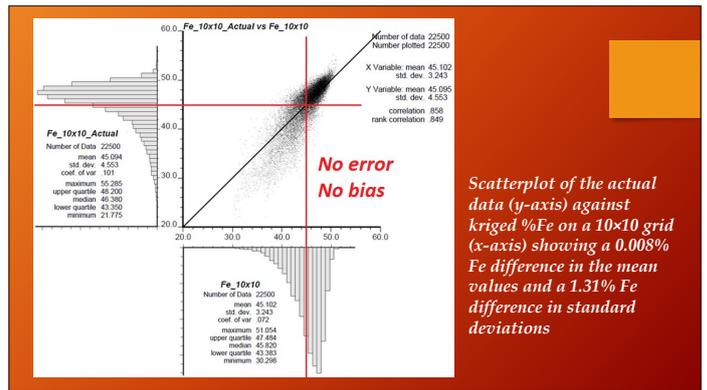



### Following that brief introduction to sampling

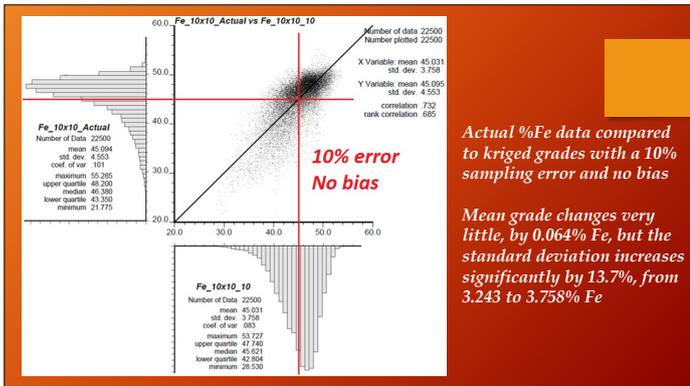
- How does **sampling bias** on grade of iron ore **affect the revenues obtained** for this product



kriged %Fe values in 10x10 grid  
 (a) with no sampling error and no bias  
 (b) with 10% sampling error only  
 (c) with 10% sampling error and 0.9x bias  
 (d) with 10% sampling error and 1.1x bias



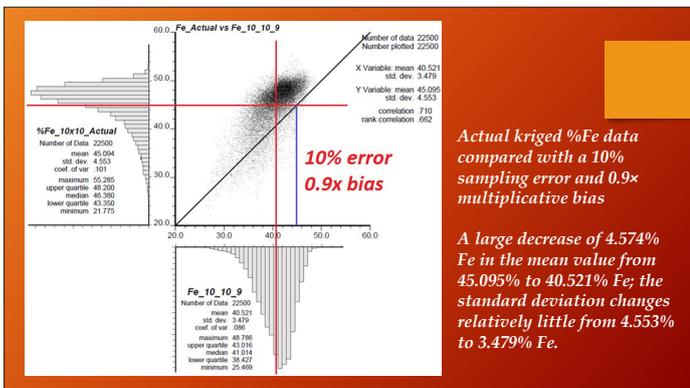
Scatterplot of the actual data (y-axis) against kriged %Fe on a 10x10 grid (x-axis) showing a 0.008% Fe difference in the mean values and a 1.31% Fe difference in standard deviations



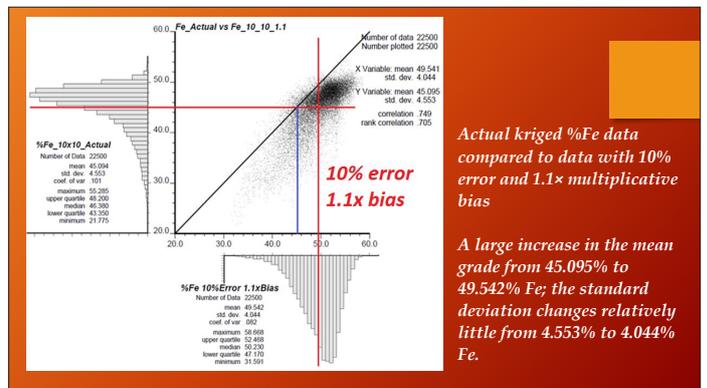
Actual %Fe data compared to kriged grades with a 10% sampling error and no bias  
 Mean grade changes very little, by 0.064% Fe, but the standard deviation increases significantly by 13.7%, from 3.243 to 3.758% Fe

Mean and standard deviation 10% error, no bias

	No error, no bias		10% error, no bias		10% error, 0.9x bias		10% error, 1.1x bias	
	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
Actual iron ore grades	45.095	4.553	45.102	3.243	45.102	3.243	45.102	3.243
Kriged iron ore grades	45.102	3.243	45.031	3.758	40.521	3.479	49.541	4.044
Difference	0.007	1.31	0.071	0.515	4.582	0.236	4.439	0.801
Percentage change	0.00016	28.77	0.0016	13.70	10.16	6.78	8.96	19.81



Actual kriged %Fe data compared with a 10% sampling error and 0.9x multiplicative bias  
 A large decrease of 4.574% Fe in the mean value from 45.095% to 40.521% Fe; the standard deviation changes relatively little from 4.553% to 3.479% Fe.



Actual kriged %Fe data compared to data with 10% error and 1.1x multiplicative bias  
 A large increase in the mean grade from 45.095% to 49.542% Fe; the standard deviation changes relatively little from 4.553% to 4.044% Fe.

Mean and standard deviation for 10% error, 0.9x bias, and 1.1x bias

	No error, no bias		10% error, no bias		10% error, 0.9x bias		10% error, 1.1x bias	
	Mean	Std dev	Mean	Std dev	Mean	Std dev	Mean	Std dev
Actual iron ore grades	45.095	4.553	45.102	3.243	45.102	3.243	45.102	3.243
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Design and Operation of Sample Cutters

- Sampling bulk commodities iron ore, manganese, chromite, bauxite, limestone, and coal, for commercial purposes is standard practice in industry
- For cutters to deliver unbiased samples is that 'all particles should have an equal chance of being sampled'

### Necessary conditions for unbiasedness

- 1) Cutter must intersect the entire stream of particles
- 2) Powered motors to drive the cutter at constant speed
- 3) Edges of cutter blades must be sharp and straight
- 4) Cutters must be able to hold all the material it extracts
- 5) No contamination of sample is permitted
- 6) Cutter blades at right angles to the trajectory of the falling stream
- 7) Vezin cutters - constant angular velocity, blades radial
- 8) Routine maintenance, cut the full stream, sufficiently wide cutter apertures, and adequately powered motors



### How will bias affect the price they receive?



- Saldanha Bay iron ore facility
- Loading 100000 tonnes of iron ore
- Taking a 21 tonne composite sample

Particle size distribution, particle volume, particle mass, mass of size fraction in increment, mass of size fraction, and possible number of particles in a 21 tonne composite sample with an average grade of 63.805% Fe

Size fractions	Diameter of largest particle	Radius	Volume of a particle (if a cube)	Mass of one particle in g	Percentage mass distribution of a typical lump ore	Mass of size fraction in sample	Mass of size fraction in gram	Possible particles in sample	Fe of size fraction	Fe of total sample
+31.5 mm	31,5	15,75	0,00001636	84	1	210	210 000	2 493	66,0	63,805
-31 +26.5 mm	26,5	13,25	0,00000974	50	7	1470	1 470 000	29 309	65,5	
-26.5 +20.0 mm	25	12,5	0,00000818	42	5	1050	1 050 000	24 934	65,0	
-25 +20.0 mm	20	10	0,00000419	22	21	4410	4 410 000	204 533	64,5	
-20 +16.0 mm	16	8	0,00000214	11	22	4620	4 620 000	418 501	64,0	
-16 +12.5 mm	12,5	6,25	0,00000102	5	18	3780	3 780 000	718 085	63,5	
-12.5 +10.0 mm	10	5	0,00000052	3	12	2520	2 520 000	935 007	63,0	
-10 +8.0 mm	8	4	0,00000027	1	7	1470	1 470 000	1 065 275	62,5	
-8 +6.3 mm	6,3	3,15	0,00000013	1	3	630	630 000	934 831	62,0	
-6.3 mm	5	2,5	0,00000007	0	4	840	840 000	2 493 352	61,5	
					100	21000	21 000 000			

### Financial impact of sampling error and sampling bias

- +25 mm fraction tends to be missed during the sampling procedure
- Due to bias in the sampling equipment assume 25% of fragments lost from the four largest fragment sizes
- 9% of the larger fragments are under-represented in the 21 t sample reducing grade by 0.10% Fe
- Large particles lost to the sample are never actually seen because they simply fall back onto the incoming stream and continue to the loading bay of the vessel

Number and mass of +25 mm particles lost during sample extraction as a result of sampling bias and the average grade of 63.705% Fe after losing the particles

Size fractions	No of +25 mm particles lost during sampling	Mass (g) +25 mm particles lost during sampling	Remaining particles	Mass left over (kg)	Percentage left over	Fe after losing particles
+31.5 mm	623	52500	1870	157,500	0,8254	63,705
-31 +26.5 mm	7327	367500	21981	1102,5	5,7775	
-26.5 +20.0 mm	6233	262500	18700	787,5	4,1268	
-25 +20.0 mm	57269	1234800	147264	3175,2	16,6392	
-20 +16.0 mm			418501	4620	24,2104	
-16 +12.5 mm			718085	3780	19,8085	
-12.5 +10.0 mm			935007	2520	13,2057	
-10 +8.0 mm			1065275	1470	7,7033	
-8 +6.3 mm			934831	630	3,3014	
-6.3 mm			2493352	840	4,4019	
Total	71453	1917300	6 754 867	19082,7	100	

### Conclusions

- Bias excludes 9% of higher grade fragments giving a grade difference of 0.10% Fe (63.805 - 63.705% Fe = 0.10%)
- Bias remains the same irrespective of the mass of the composite sample
- The 0.10% Fe bias in the grade for a 100 000 t load at a price of \$70 per ton and the lot grade of 63.805% Fe would amount to a loss of \$10 971, not much on a load worth \$7.0 million
- South Africa exports 60 Mt of iron ore on 600 ships annually
- Cumulative losses per annum could be as much as \$6,6 million

### Acknowledgements

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