

Theory and Practice of Particulate Sampling: an Engineering Approach

Geoffrey J. Lyman BEng, MEng, PhD

Materials Sampling & Consulting Pty Ltd. materials-sampling-and-consulting.com



A full development of particulate sampling theory on a sound mathematical basis that goes beyond the work of Gy and others
This is a text for people who are serious about sampling system design and quality control of these systems

The Theory of Sampling (TOS) as put forward by Pierre Gy forms a foundation for the sampling of particulate materials and has been in use now for many years, especially in the mineral industries. It is only recently that interest and appreciation of sampling theory has found its way into some corners of the mineral industry, such as industrial minerals, other industries and the power of the theory has been recognised and harnessed.

One aspect of the statistical theory of sampling that has been lacking is the ability to calculate the entire sampling distribution. The result of a sampling procedure or protocol is a numerical result. This is a statistical quantity because it is subject to uncertainty; statisticians call this a random variable. To know everything about a statistical quantity, it is necessary to know its distribution, which is quantified by its probability density function or distribution function. Until the author's developments, statistical sampling theory provided only the variance or "spread" of the distribution of the sampling uncertainty. When dealing with sampling of low and trace level components of a mixture, such as precious metals, mycotoxins and valuable mineral contents of process streams such as tailings or waste streams, the distribution of the concentration of the target constituent is usually skewed (asymmetric). This is almost always the case with gold bearing materials, and there has been great difficulty in the correct and successful application of sampling theory to gold ores.

To overcome this restriction, the author has developed the whole of particulate sampling theory from the premise that the numbers of particles, or grains of mineral of any one type in a mixture, follow Poisson distributions. This assumption alone permits all of the mathematical results of sampling theory to be derived in a completely coherent manner. The resulting theory encompasses all previous correct presentations

of sampling theory and takes the theory to the point where the entire sampling distribution can be calculated. This novel accomplishment is of particular value to the gold industry where the impact of coarse gold on sampling uncertainties can be profound and have serious economic impact.

With the publication of this text, statistical sampling theory for particulate materials has been brought to a full conclusion and end point. If a source for the teaching of statistical sampling theory is required, this text provides all that is required. So-called intrinsic or constitutional heterogeneity is completely dealt with. The text provides means of quantifying the heterogeneity of particulate material either through laboratory work or the use of the scanning electron microscope. The text also deals fully with sampling variance due to time- or tonnage-wise grade variation, which is known as distributional heterogeneity.

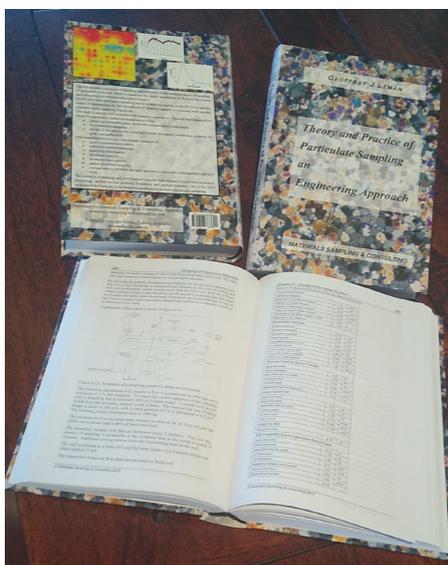
The text is fully self-contained. Even though there is a good deal of mathematical

and statistical detail in the book, a comprehensive Appendix dealing with all aspects of statistics in the book is provided so that the reader can rapidly come up to speed with the material required to fully understand and utilise all material.

The text includes a chapter dealing with the design of sampling systems for a gold run of mine ore, a raw coal sampling system and for a hard to handle damp bauxite ore. The engineering details of the system designs are considered with quantitative calculations of sampling uncertainties and material flows. Practicing mineral process engineers will find these examples of particular value.

Apart from the uncertainties introduced by the particulate nature of minerals and other material when sampling, it is also vital to understand the uncertainties that stem from the variation of material grade with time or tonnage. The uncertainties involved in sampling in process settings such as in mineral processing plants are almost always dominated by the uncertainties due to these variations (distributional heterogeneity) and are analysed mathematically by the methods used by geostatisticians when dealing with spatial grade variations in ore bodies. The basis of such analyses is the theory of random functions and the principal tool used is the variogram or covariance function which characterises the time-wise or tonnage-wise variation in a process stream. Gy developed his work at a time that geostatistics as a discipline was just appearing. Gy's inclusion of variogram methods was very much simplified as a result and with the further development of geostatistics and computing power, we can do much better today.

The text treats variograms in great detail and provides a variety of variogram models that can be fitted to data by both conventional methods and the improved method of maximum likelihood. A comprehensive



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Appendix deals with variograms and includes methods of simulation of random functions which is vital for anyone wanting to build a total simulation of a sampling system.

A major chapter deals with the types of mechanically correct sampling equipment available for the plant and the preparation of samples down to the aliquot level. Methods of modern on-line analysis are also treated with the inclusion of methods of determination of the precision of such analysers with examples.

The text does not forget that sampling of processes leads to laboratory analysis of the samples with the concomitant issues of quality control which is dealt with in a separate chapter and shows how modern cumulative sum techniques can be used to improve upon the standard approach of statistical quality control with Shewhart charts. A chapter deals with analytical uncertainties in laboratories.

A chapter is also devoted to methods of statistical material balancing (metallurgical accounting) where the author has wide experience.

For the reader from outside the mineral processing field, a chapter deals with the sampling of foodstuffs, including the difficult sampling of grain for mycotoxins, the sampling of packaged material and environmental sampling for site contaminants. This material is not found elsewhere.

A number of other issues are dealt with in the text that will be helpful to the reader dealing with sampling systems on a routine basis. The topics of mixing and blending,

bias testing and the method of maximum likelihood, which is perhaps the most powerful statistical analysis tool ever invented, are presented.

It seems that many people express an interest in sampling theory and its practical implementation and take courses on the subject either at conferences or through in-house training by a sampling authority. However, in the author's experience, few actually get to the point where they can take on a sampling problem in full. Part of the problem stems from the fact that statistics is a "nasty" subject and soon (gratefully) forgotten from university studies.

The author is a widely experienced chemical and mineral processing engineer who has had a research career solving problems through the application of mathematics and statistics. The title of the text suggests that the book will appeal to engineers; it was written with the intent that it could be used by engineers to create systems that achieved "representative" sampling and analysis. The word "representative" really means that the results from the sampling scheme are "fit for purpose". The result delivered is accurate (unbiased) and sufficiently precise that use of the result does not expose the user to unacceptable financial risk. The text will permit you to achieve this goal.

The text is available through the author's website. The website will also provide the opportunity to access software implementing methods of analysis described in the text. The MSC Toolbox consists of applications that permit rapid calculation of variograms from detrended data as well as

fitting of a wide variety of variogram models to data. You will be accessing the tools that have been developed over nearly 40 years of expediting the solution of problems for clients.

The author has stepped away from the viewpoint on sampling theory first put forward by Pierre Gy and has derived and extended sampling theory by taking a rigorous approach using mathematical statistics. This results in a simplification of the notation and presentation of the theory. The new approach permits the extension of the theory to the point where the exact sampling distribution as well as all the moments of the sampling distribution, not just the variance, can be calculated. An appendix of background statistical information has been included to assist readers in understanding the mathematical procedures used throughout the text. The practical aspects of sampling are well covered with examples of the design of sampling systems for gold, bauxite and coal. The text also covers the sampling of foodstuffs such as bulk grains and packaged foods. Sampling in these circumstances involves random distribution of contaminants throughout a lot and requires special statistical models outside of current sampling theory to deal with this type of distributional heterogeneity. Certain cases of environmental sampling are also included. This text will be of use to engineers, food scientists and regulators in understanding sampling of particulate and packaged materials and can form a complete text for the teaching of sampling theory for commodities.

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