Comminution sizes in sampling calculations

D. Francois-Bongarcon[a] and R. Oliver[b]
[a] **AGORATEK International Consultants Inc. (AICI) E-mail: dfbgn2@gmail.com**
[b] **formerly Intertek Laboratory, Jakarta**

As was demonstrated by Gy empirically, then elucidated by Francois-Bongarcon, only the P95 size of comminuted material (i.e. the size of a screen rejecting 5% of the material in mass) is relevant to sampling theory variance formulas. In practice, when establishing sampling nomograms or using historic data for heterogeneity testing, the P95 is not always known and the material is not always preserved for P95 determinations by screening. A non-theoretical study of the experimental material previously used and published by Gy had suggested some unpublished, rule-of-thumb, order-of-magnitude correspondence between other Pxx sizes (e.g. P100, P80, P75, etc.) and P95, to be used as a last resort when no better determination is possible. This correspondence surprisingly followed an arithmetic progression. In this paper, various types of size distributions were researched and most of these rule-of-thumb formulas were reasonably confirmed, except for the correspondence between P95 and P100 (maximum size), which was therefore updated. The reasons for the arithmetic progression and its exception for P100 also became clearer in the process.

Introduction

In the process of establishing a numerical model (formula) for the relative variance of the fundamental sampling error, Gy stumbled on two problems: defining the comminution size using a single size parameter (nominal size) and relating it to the average fragment volume weighted by mass, in the lot to be sampled. In his well known formula:

$$\text{Rel.Var.} = cfg\ell\, d^3\, (1/M_S - 1/M_L) \qquad (1)$$

indeed, the term fgd^3 intends to represent that average quantity. It includes the chosen nominal size d, a shape factor f that transforms the cube of that size into a volume, and the granulometric factor g, which pretends to transform the nominal volume fd^3 into the mass-weighted average volume in the lot.

This feat was not obvious from the start. Defining the comminution degree using a single parameter had been done before using a "percent passing" size, and Gy decided not to innovate there. But which percentage passing was to be used? And how could it be miraculously related to the average volume in the entire distribution of fragments, irrespective of the type of material and comminution, using a simple multiplicative and universal constant g? As it turned out, it is indeed possible, but only if we restrict the definition of the comminution degree to the use of a 95% passing size (a.k.a. P95) for d and use an approximate value of 0.25 for constant g. That has been described and demonstrated [1].

- Since then, two practical problems have arisen recurrently:
- Should laboratories be coaxed, often against their will, to use only P95 size on their comminution standards and controls?

When performing heterogeneity testing, i.e. model (1) calibration, or when establishing or optimising preparation protocols and sampling nomograms, what if the P95 size is not available and no material left that can be screened (a typical situation when dealing with historic data)?

Practical derivations of P95 to date

Going back briefly to Gy's purely experimental study of that constant g, Francois-Bongarcon established rules of thumb (ROT) to derive the likely P95 size when only another Pxx size is known. This was done by identifying a subset the typical curves, out of the well known 114 experimental used by Gy to relate the percent reject x (i.e. 100% - xx%) to the mass-weighted average volume of the fragments [2]. This subset was selected so as to represent the types of materials and comminution most often encountered in process plants and laboratories. These unpublished rules, which have been used as required but successfully since circa 2000, are shown on Table 1. They revealed a curiously arithmetic progression.

doi: https://doi.org/10.1255/tosf.140

Table 1. ROT for P95 (circa 2000)

xx(%)	P95/Pxx
95	1.00
90	1.25
85	1.50
80	1.75

That left the problem of 'guestimating' P95 when only P100 (a.k.a. Pmax or d-max) is known. Indeed it is not rare in protocols to prescribe crushing to a maximum size, which is usually achieved using a screen and iteratively returning its oversize to the crusher until all the material passes. It was in slightly more recent times that Francois-Bongarcon thought to have found a way to relate P95 to the maximum size.

Indeed, Gy had also published a curve (reproduced as Figure 1) that showed an alternative granulometric factor g' as a function of the ratio r=d-max/d-min. This new, rigorously demonstrated factor g' must be substituted to constant g in formula (1) in such a case, applying it to d-max^3 instead of d^3 (where $d = d_{95}$ = P95).

The study and mathematical formula that yielded that curve was not initially found. It was noticed by Francois-Bongarcon [3], however, that the asymptote for very large ratios seemed to be close to 0.1. Assuming this was true, and observing uncalibrated material would correspond to an infinite value of the ratio r, to the limit, its gd_{95}^3 would also be equal to g'd-max^3 with a g' taken as the apparent asymptotic value of 0.1. Curiously again, this led to P95 = 0.75 d-max, which could hardly be another coincidence, so that a new table of ROT was used since then that included P100 (Table 2).

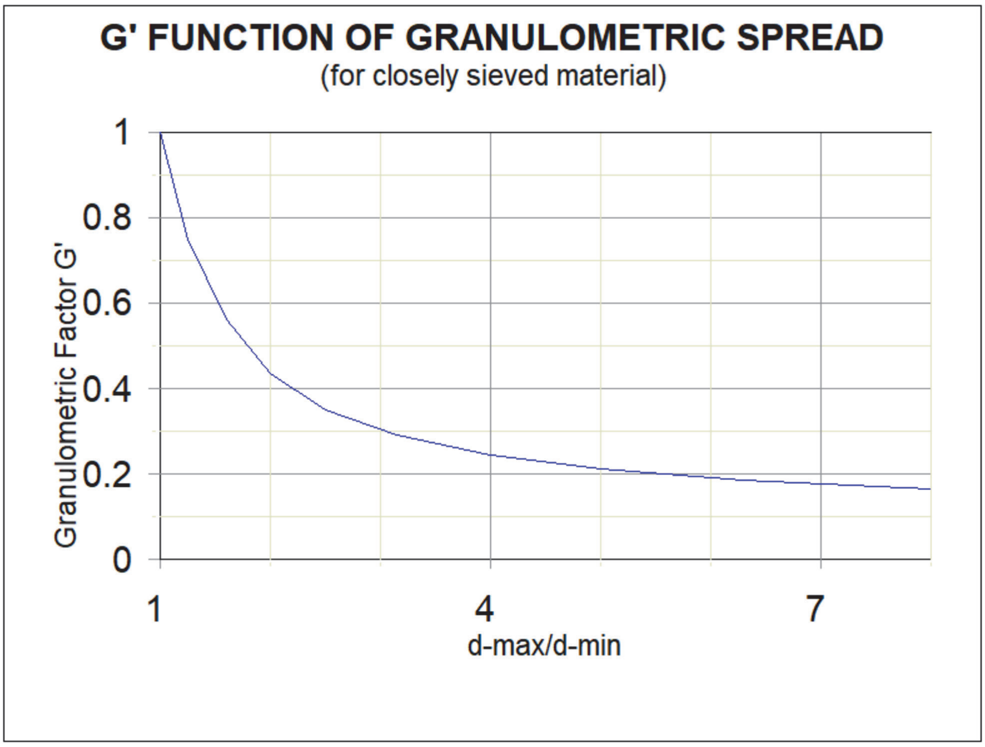

Figure 1. Granulometric factor for closely sieved material

Table 2. Revised ROT for P95 (published 2019)

Xx(%)	P95/Pxx
100	0.75
95	1.00
90	1.25
85	1.50
80	1.75

Experimental fact checking

The time had come to (re-)verify these handy figures experimentally, especially the relationship to P100, as part of a series of papers being written on size distribution models.

After numerous discussions of the topic with the Intertek Laboratories in Jakarta, three usable data sets consisting of 10 samples each with detailed size distributions by screening were obtained. Two sets were for pulverized ore material, the third one was for crushed material. For each of the sets, it was possible to interpolate the Pxx for xx=5, 10, 15 and 20. A Rosin-Rammler model was used to improve the interpolations (over a linear one). For each sample of the crushed material, the maximum fragment size (P100) was evaluated based on screen retained masses by linear extrapolation, which was confirmed best over other methods, in the case of crushed material, using photos of the coarsest fragments. In each case, the P95/Pxx factors were calculated. The averages of these are shown in Table 3. They confirm the older ROTs, except for P95/P100, for which the new value of 0.40 is now recommended instead of 0.75.

This new, experimental confirmation of reasonableness the old ROTs for P90 and below make sense since they were already established based on Gy's experimental results. In retrospect, that the new experimental value of P95/P100 does not lie in the linear progression of the others makes sense. Cumulated size distribution curves have a stronger inflection/curvature somewhere above 95%, the strength of which, under the blinding charm of completeness of the observed linear progression, had been overlooked (Figure 2 – with any x-scale and/or units).

Table 3. P95/Pxx Factors and ROTs

	p95/pmax	p95/p95	p95/P90	p95/p85	p95/p80
Crushed	0.47	1.00	1.33	1.73	2.05
Pulps	0.42	1.00	1.27	1.47	1.72
pulps(2)	0.36	1.00	1.20	1.37	1.50
AVG	0.42	1.00	1.27	1.52	1.76
Min	0.26	-	1.11	1.20	1.33
Max	0.64	-	1.74	2.10	2.55
Initial RoT	*0.75*	1.00	1.25	1.50	1.75
New RoT	**0.40**	1.00	1.25	1.50	1.75

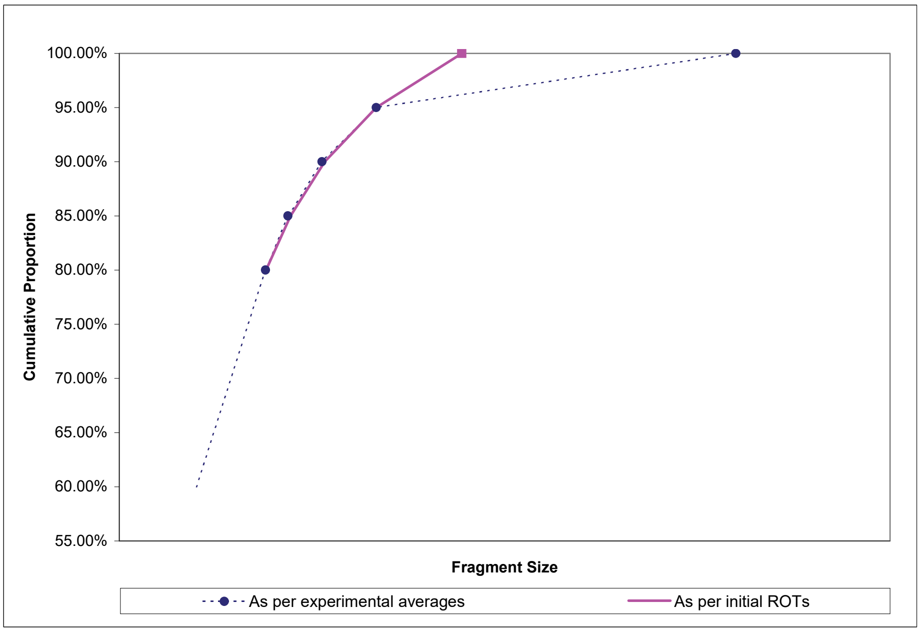

Figure 2. Average Size Distribution of the Material

Closely sieved material

The curve of Figure 1 above was derived from a formula found in section 55.4.2 of Gy,1967[2], where the complete study of the g' substitute to the classical granulometric factor g is described.

Gy started off assuming that material was sieved between two screens of mesh d_0 (=d-max) and d_R (=d-min), separated by a number 'R' of AFNOR standard screens sizes, or 'modules'. The AFNOR (French National Standardisation Organisation) screen sizes are in geometric progression of ratio $10^{1/10}$ which is very close to $2^{1/3}$, so that:

$$d_R^3 = d_0^3 / 2^R \qquad (2)$$

He then set up to estimating the average mass-weighted volume of the calibrated fragments between the two screens. Assuming approximately uniform distribution of masses in throughout the R modules, he found that the average cubed fragment in the calibrated material could be written $g'd_0^3$ where:

$$g' = (1.5 - 1/2^{R-1} + 1/2^{R+1}) / R$$

If we call r the ratio d-max/d-min = d_0 / d_R , by virtue of (2),

$$R = 3\ LN(r) / LN(2)$$

which allows us to adequately calculate g' from r, as on Figure 1, independently for the AFNOR progression.

As it turns out, the asymptote of that curve, however slowly reached, therefore is zero. This clearly invalidates any previous asymptotic reasoning to derive P95 from P100, which explains the wrong ROT for P100 that had been previously found.

Conclusion

The experiment described in this paper is leading to the following take-away, but one should remember that deriving a P95 using ROTs remains a last resort when direct measurements are not possible:

- Previous ROTs below P95 were confirmed
- The ROT concerning P100 was found to be erroneous and was corrected
- The New ROTs in Table 3 are now believed to be safe to use

In particular, given the natural variability of comminution results from sample to sample, it may not be necessary to force laboratories to change their definitions of comminution standards and controls, as long as sampling calculations and nomograms are always made using P95 equivalents derived with these ROTs. This important conclusion is novelty.

References

1. Francois-Bongarcon, 1998. Extensions to the Demonstration of Gy's Formula. in Proceedings CIM/CMMI/MIGA Montreal'98 Conference.
2. Gy, P.M. 1967. L'Echantillonnage des Minerais en Vrac, Tome 1. in Mémoires du BRGM, No. 56. Editions BRGM. Paris. 189.
3. Francois-Bongarcon, D. 2019. The Liberation Factor And Other Parameters: Are we at the end of the journey? in Proceedings Ninth World Conference on Sampling and Blending, Beijing.